

# An Effective Methodology with UPFC Controller for Locational Marginal Pricing in Competitive Energy Markets

P. Vidhya, R. Ashok Kumar, K. Asokan

**Abstract**— Deregulation and privatization of power market worldwide has forced to identify the different ancillary services and the service providers in order to price the services properly. Active and reactive power pricing is an important process that can never be avoided and constitutes a considerable part of economy. An exclusive reactive power management is essential for the secure operation of power systems and reactive power support is considered as an essential system support service for competitive electricity power markets. In this market, Pool-co and bilateral power trading plays a vital role where the amount of power and the flow path between the transacting generators and loads are fixed beforehand. The ability of FACTS devices is to control the power flow through designated routes in transmission lines and thereby reducing the overloading of lines and ensures the more flexible operation. The objective of this paper is to propose various transmission pricing approaches with FACTS devices of Unified Power Flow Controller (UPFC) for determining the locational marginal prices of proposed test system. A case study with Indian 246-NREG bus system is conceded to illustrate the effectiveness of the proposed transmission cost allocation procedures and test results are presented. Finally, the simulation results are compared with/without FACTS devices for Pool-co and bilateral energy markets.

**Index Terms**— Real and Reactive power, Marginal price, Reactive power cost model, FACTS cost model, Pool model, Bilateral model.

## I. INTRODUCTION

The electricity supply industries all over the world is alight upon restructuring their electricity business into competitive environment for better utilization of the resources, technological innovation, quality of service and adequate better choice to the consumers at competitive prices. Electricity sector deregulation, also known as restructuring is expected to attracts investment, promote efficiency, increase technical growth and improve better operation of the system [1].

In this environment, transmission system plays a substantial role in competitive electricity markets. Reactive power service is one of the key issues of ancillary services and it paves the way for better transaction of power in electricity markets. The reactive power service is essentially required for transmission of active power, voltage control and reliable operation of the systems in the competitive electricity market

structure. It is revealed that establishment of accurate price of reactive power can able to recover the production cost of reactive power and also to provide system voltage support. The prolonged research on transmission pricing shows that, there is no generalized theory on pricing methodology. In general, each electricity market has adopted a method based on particular topology of the network [2].

Many investigations and several studies have been carried out of proper method of reactive power pricing. In order to maintain the good voltage profile and preserve system operational reliability, the spot price of reactive power has gained importance and should be given much attention [3].

Most of the researchers have been focused on real power transactions as the important one. In some systems, the reactive power cost is included in the price of active power pricing of real and reactive power [4]. In ref [5], the authors analyzed the reactive charging scheme composed of recovering capital cost and operating cost. Locational spot prices for reactive power could provide adequate incentives for loads to consume reactive power and for generator to produce reactive power sufficiently.

In ref [6,7], the authors developed a model which involves reactive power pricing and revealed that the use of lagrangian multipliers in OPF represents the marginal costs of the node power injections and the non- linear reactive power optimization problem was solved by successive LP method. The determination of wheeling marginal cost of reactive power was described in ref [8].

In 1990, a simple approach has been developed for reactive power planning which combines the issue of reactive power pricing, is used to recover the cost of installed capacitors by using OPF approach [9]. The simulations results and theory of real time pricing of real and reactive power using a social benefit was explained in ref [10]. The modifications were made in OPF algorithm summary in reactive power pricing and it's features are presented in ref [11]. Recently, a new approach has been formulated for calculating the reactive power production cost by non-linear model representation, which includes the detailed model of heating limits of the armature and field under the excitation limit [12]. The modified OPF was presented with sequential linear programming technique with a interior point method used for calculating active and reactive power marginal prices [13].

In recent years, the present pace of power system restructuring faces a large demand of electricity problem has been placed on the transmission network and demands will continue to increase due to increasing number of non- utility generator and intensified competition among them. More advanced technology is provided for secure and reliable operation of transmission and distribution in power system

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To increase the power transfer capability and achieve better utilization of existing power system, the flexible AC transmission system (FACTS) controller have become imperative. FACTS controllers have the capability of direct-line control of transmission line flows by changing the transmission line parameters such as line impedance and power angle of transmission corridors [14].

In ref [15], the authors proposed that installation of FACTS controllers with their optimal location can change the power flow pattern stability, security, reliability and economic efficiency of the system by changing the wheeling cost of power due to impact on nodal price of real and reactive price and therefore these FACTS devices cost functions should also be incorporated in an objective function which provide noticeable changes in nodal prices of both real and reactive power.

Olivera et al., suggested that allocation of FACTS devices and their domination in transmission pricing was presented in ref [16]. The impacts of SVC and TCSC on the spot prices of real and reactive power were determined and maximizing the social welfare function are studied in [17].

The effects of optimally located SVC and TCPAR on the real and reactive power price includes the costs of FACTS controller has been described in ref [18]. In the restructured environment the number of bilateral transactions has grown rapidly and it is essential to help the system operator to evaluate their impacts on system operation and impacts on nodal price determination [19]. Singh and David et al., introduced the concept of optimal location of FACTS device which is determined using line power based sensitivity index, performance based index, loss sensitivity based index, price based index with inclusion of FACTS parameters [20]. In ref [21], the authors ascertain the locational marginal prices with SVC controller for pool and hybrid market model.

In this article, nodal prices have been computed for pool and bilateral transactions by considering of three different reactive power cost model for generator's reactive power cost calculation. The Simulation result has been done in two parts. The first part includes the numerical approach without considering FACTS devices and the second part includes the FACTS devices in the system. The impacts of FACTS controller have been incorporated taking their cost functions into account. The proposed approaches have been tested on Indian 246-bus NREG system and the comparisons have been given for different reactive power cost model of pool and bilateral model to illustrate the superior performance of the proposed system.

## II. VARIOUS MODEL IN ELECTRICITY MARKET:

In the deregulated electricity market structure, the different transactions may takes place either directly or indirectly between sellers and buyers because the market is under competition and hence it becomes an open access. Based on the transactions, the electricity market model is modeled based on their mode of transaction. There are three major model of transaction of power in electricity market structure [22, 23].

1. Pool Co Model
2. Bilateral Model
3. Hybrid Model

### Pool Co Model:

The pool co model is defined as the centralized market place which clears the market for sellers and buyers. The power sellers and buyers submit their bids to inject power in to and out of the pool. In this model, only single entity called system operators plays a major role to have the contract between the retailer and consumers. The low cost generator would especially reward in this model.

### Bilateral Contract Model

This model is referred as the direct access model because this model permits the direct contracts between the power producers and the consumers without entering in to pooling arrangement. The establishment of non-discriminatory access and the pricing rules for the transmission and distribution systems the direct sales of power takes place between the utilities are guaranteed.

The bilateral contract model may also include some other transaction such as:

### Bilateral Transactions

A bilateral transactions means there is a direct transaction between the power producers and the customers.

### Multilateral Transactions

Multilateral transactions are the extension of bilateral transactions and the trading arrangement is done by energy brokers with two or more parties.

### Ancillary Service Transactions

Ancillary services are defined as all those activities such as regulation of frequency and tie-line power flows, voltage and reactive power control and ensuring system reliability and maintain secure operation of the system. To provide the essential ancillary services for system regulation, the system operator (SO) may arrange some direct transactions with some of the generation companies (GENCOs). The ISO has the major role in this transaction and simply dispatches all transactions and charges for the service.

### Hybrid Model

It is a combination of both pool co model and bilateral model. In this model, trading takes place between the group of sellers and buyers and the consumers and producers has the choice of selection in any model.

### Mathematical Description of Bilateral Contract Model

The bilateral contract model used in this work is basically a subset of the full transaction matrix  $T$  proposed in [24]. Its general concept is mostly composed as a multimode case, where the seller from the Generation Companies ( $GENCO_s$ ) and buyer from distribution companies ( $DISCO_s$ ) are involved in the process. The transaction matrix  $T$  is a collection of all possible transactions between Generation ( $G$ ), Demand ( $D$ ) and any other trading entities ( $E$ ) such as marketers and brokers and it is shown in equation (1).

$$T = \begin{bmatrix} GG & GD & GE \\ DG & DD & DE \\ EG & ED & EE \end{bmatrix} \quad (1)$$

It is assumed that entire transactions activities are employed between  $GENCO_s$  ( $G$ ) and  $DISCO_s$  ( $D$ ). There is no contract made between the two suppliers or two consumers. Hence it is

noted that, the diagonal block matrices (GG and DD) are considered as zero. Hence neglecting transmission losses, the transaction matrix can be simplified as:

$$T \equiv [GD] = [DG^T] \quad (2)$$

Where GD and DG represents the bilateral transaction between GENCOs and DISCOs.

From that, each element of transaction matrix T namely  $t_{ij}$  represents bilateral contracts between suppliers ( $p_{gi}$ ) of row i with a consumer ( $p_{dj}$ ) of column j. Then the sum of row i represents the total power produced by generator i and sum of column j represents the total power consumed at load j.

$$T \equiv \begin{bmatrix} t_{1,1} \dots & t_{1,nd} \\ t_{2,1} \dots & t_{2,nd} \\ \vdots & \vdots \\ t_{ng,1} \dots & t_{ng,nd} \end{bmatrix} \quad (3)$$

Where  $n_g$  and  $n_d$  represents generators and loads respectively. Based on the conventional load flow variables, the generation  $p_g$  and load  $p_d$  vectors can be expanded in two dimensional transaction matrix T as given in equation (4).

$$\begin{bmatrix} P_d \\ P_g \end{bmatrix} = \begin{bmatrix} T^T & 0 \\ 0 & T \end{bmatrix} \begin{bmatrix} u_g \\ u_d \end{bmatrix} \quad (4)$$

In the above matrix equations, the  $u_g$  and  $u_d$  are column vectors of ones with the dimension of  $n_g$  and  $n_d$  respectively. There are some intrinsic properties which are associated with the matrix (T) and these properties have been explained in [25]. Each contract has the range originates from zero to maximum allowable value  $T_{ij}^{\max}$ . This maximum value is bounded by the value  $P_{gi}^{\max}$  or  $P_{dj}^{\max}$  whichever is smaller. Hence the range rule satisfies the following equation.

$$0 \leq T_{ij} \leq T_{ij}^{\max} \leq \min(P_{gi}^{\max}, P_{dj}^{\max}) \quad (5)$$

There is a possibility for some contracts to be firm, so that  $T_{ij}^0$  is equal to  $T_{ij}^{\max}$ . According to flow rule, the line flows of the network in ac model can be formulated as follows:

$$P_{line} = ACDF [P_g - P_d] \quad (6)$$

The AC distribution factors (ACDFs) is defined as the change in real power flow ( $\Delta p_{ij}$ ) in a transmission line-k connected between bus-i and bus-j due to unit change in power injection ( $\Delta p_n$ ) at any bus-n.

Mathematically, the matrix ACDF, for line-ij can be written as

$$ACDF_n^{ij} = \Delta p_{ij} / \Delta p_n \quad (7)$$

The matrix ACDF is the distribution factors matrix which is computed using AC load flow technique [26]. The representations of  $p_{gb}$  and  $p_{db}$  are substituted by using the definition of T as given in equation (4) and the line flows obtained for bilateral transaction can be expressed in other way as:

$$P_{line} = ACDF [T - T^T] \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \quad (8)$$

The matrix ACDF is based on Jacobin sensitivity matrix and it includes any changes in the system operating conditions.

$$GD_{sb}^{\max} = \min(P_{GB, sb}^{\max}, P_{DB, bb}) \quad (9)$$

### III. MATHEMATICAL FORMULATION FOR NODAL PRICE DETERMINATION WITH REACTIVE POWER COST MODEL OF GENERATORS

In this paper, the optimization problem is solved by minimizing total cost subject to equality and inequality constraints for pool electricity market model by including real and reactive power nodal prices, fuel cost, cost components of reactive power with different cost model and Facts devices.

#### Objective function:

The objective function can be represented as:

$$\text{Min TC} = \sum_{i=1}^n \text{Cost}(P_i) + \text{Cost}(Q_i) + \xi_{UPFCi} * \text{Cost}(F_i) \quad (10)$$

The objective function consist of three cost components such as cost of real power, cost of reactive power and cost of FACTS devices.

Let

Cost ( $P_i$ ) = Cost function of real power for NG (No of generators)

Cost ( $Q_i$ ) = Cost function of reactive power for set of NG generators

Cost ( $F_i$ ) = Cost function of FACTS devices (UPFC).

where:

$$\text{Cost}(P_{Gi}) = a_p P_{Gi}^2 + b_p P_{Gi} + c_p \quad \$ / h \quad (11)$$

$$\text{Cost}(Q_{Gi}) = a_p Q_{Gi}^2 + b_p Q_{Gi} + c_p \quad \$ / h \quad (12)$$

#### Operating constraints:

(i) The real and reactive power flow equations from bus-i to bus-j can be written as:

$$P_{ij} - V_i^2 G_{ij} - V_i V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) = 0 \quad (13)$$

$$Q_{ij} - V_i^2 (B_{ij} + B_{sh}) - V_i V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) = 0 \quad (14)$$

(ii) The real and reactive power flow equations from bus-j to bus-i can be expressed as:

$$P_{ji} - V_j^2 G_{ij} - V_i V_j (G_{ij} \cos \delta_{ij} - B_{ij} \sin \delta_{ij}) = 0 \quad (15)$$

$$Q_{ji} - V_j^2 (B_{ij} + B_{sh}) + V_i V_j (G_{ij} \sin \delta_{ij} + B_{ij} \cos \delta_{ij}) = 0 \quad (16)$$

$$S_{ij} = \sqrt{P_{ij}^2 + Q_{ij}^2} \leq S_{ij}^{\max} \quad (17)$$

(iii) Real and Reactive Power generating limits

$$P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max} \quad (18)$$

$$Q_{gi}^{\min} \leq Q_{gi} \leq Q_{gi}^{\max} \quad (19)$$

(iv) Real and Reactive Power Balance constraints

$$\sum_{i=1}^{Ng} P_{gi} - P_{di} - P_{loss} = 0 \quad (20)$$

$$\sum_{i=1}^{Ng} Q_{gi} - Q_{di} - Q_{loss} = 0 \quad (21)$$

(v) Power Flow Constraints

$$P_{ijmin} \leq P_{ij} \leq P_{ijmax} \quad (22)$$

$$Q_{ijmin} \leq Q_{ij} \leq Q_{ijmax} \quad (23)$$

(vi) Voltage Magnitude Limits

$$V_{imin} \leq V_i \leq V_{imax} \quad (24)$$

(vii) Voltage angle limits

$$\delta_{imin} \leq \delta_i \leq \delta_{imax} \quad (25)$$

(viii) Reactive Power Capability

Curves limit for generators:

$$P_G^2 + Q_G^2 \leq (V_i l_a)^2 \quad (26)$$

In case of hybrid market model, additional constraints to be satisfied are:

(ix) Equality constraints for bilateral transactions using transaction matrix  $GD$  are expressed as follows:

$$P_{DB} = \sum_{i \in sb} GD_{ij} \quad (27)$$

$$P_{GB} = \sum_{j \in bb} GD_{ij} \quad (28)$$

$$P_g = P_{GB} + P_{GP} \quad (29)$$

$$P_d = P_{DB} + P_{DP} \quad (30)$$

$$P_{fb} = ACDF (P_{GB} - P_{DB}) \quad (31)$$

$$P_{fp} = ACDF (P_{GB} - P_{DP}) \quad (32)$$

$$P_f = P_{fb} + P_{fp} \quad (33)$$

$$GD_{sb}^{\max} = \min(P_{GB, sb}^{\max}, P_{DB, bb}) \quad (34)$$

(x) Limit on FACTS controllers:

UPFC:

$$-u \cdot \phi_T^{\max} \leq \phi_T \leq u \cdot \phi_T^{\max} \quad (35)$$

$$-u \cdot I_q^{\max} \leq I_q \leq u \cdot I_q^{\max} \quad (36)$$

$$0 \leq V_T \leq u \cdot V_T^{\max} \quad (37)$$

$u$  is the vector of binary variable ('0's and '1's) representing the presence or absence of UPFC. It is assumed that '1's represent presence and '0's represent absence of FACTS devices.

### Various approaches of Reactive Power Cost Model

Generally the cost of reactive power produced by a generator is essentially composed of two components namely: fixed cost also called as investment cost and variable cost. The variable cost in turn consists of opportunity cost which includes fuel cost and maintenance cost. The opportunity cost is imposed on the generator resulting from reduction of its active power generation [27]. The three methods have been considered for calculating the cost of reactive power of generators.

### Triangular Approach (Method 1):

To overcome the draw backs associated with conventional cost methods, the researchers in 2005, proposed a method for evaluation of reactive power based on a triangular relationship between active and reactive power. In this method, the reactive power cost calculation is essentially composed on the formulation of active power cost, in which the active power is replaced by reactive power using the triangular relationship [28]. In this triangular approach, the cost of reactive power is formulated as follows:

$$\text{Cost}(Q) = a''Q^2 + b''Q + c'' (\$/\text{hr}) \quad (38)$$

From the power triangle the constants  $a''$ ,  $b''$ ,  $c''$  are calculated depending on power factor ( $\cos \theta$ ) and are calculated as follows:

$$a'' = a_p \sin 2\theta$$

$$b'' = b_p \sin \theta$$

$$c'' = c_p$$

### Maximum Real Power Based Approach (Method 2):

In this approach, if the generator produces its maximum active power ( $P_{\max}$ ), then its cost for generating the active power is ( $P_{\max}$ ). Hence in this situation, no reactive power is produced and therefore,  $S$  equals  $P_{\max}$ . The production of reactive power itself does not seem to impose any fuel cost on generator except the losses. Hence, reactive power production by a generator will result in reduce its capability to produce its active power. To generate reactive power  $Q_i$  by considering generator  $i$  which has been operating its nominal power ( $P_{\max}$ ), it is required to reduce its active power to  $P_i$  such that

$$P_i = \sqrt{P_{\max}^2 - Q_i^2}, \Delta P = P_{\max} - P_i \quad (39)$$

Where  $\Delta P$  represent the amount of active power that will be reduced as the result of generating reactive power. The cost of reactive power  $Q_i$  is precisely calculated by imposing the following cost components.

To accurately calculate the cost of reactive power  $Q_i$ , the following cost imposed on generator is given below:

Cost ( $P_{\max}$ ): Cost of producing active power ( $P_{\max}$ ) in an hour.

Cost ( $P_{\max} - \Delta P$ ): Cost of generator when producing both active and reactive power with the amounts  $P_i$  and  $Q_i$ , respectively.

Cost( $P_{\max}$ ) - Cost ( $P_{\max} - \Delta P$ ): It is represent as the reduction in the cost of active power due to compulsory reduction in active power generation ( $\Delta P$ ) which is useful to generating reactive power with the amount of  $Q_i$ . This represents the cost of reactive power production while the operating point of generator is moved from point 1 to point 2 (**Figure 1**) as below:

$$\begin{aligned} \text{Cost}(Q_i) &= \frac{P_{\max} - \Delta P}{P_{\max}} \text{Cost}(P_{\max}) \\ &- \text{Cost}(P_{\max} - P_i) \quad \$/\text{hr} \end{aligned} \quad (40)$$



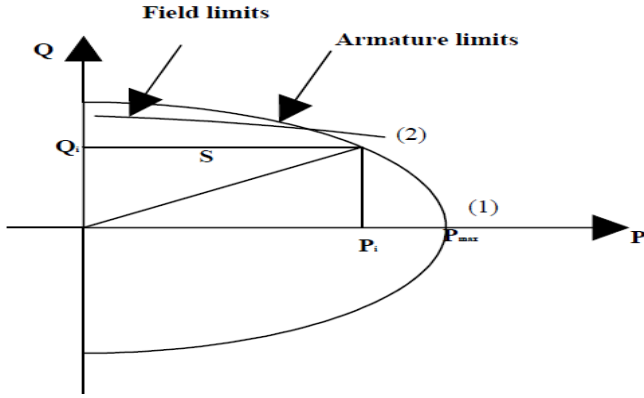


Figure 1. Capability curve of generator.

#### Maximum Apparent Power Based Approach (Method 3):

The reactive power generation capability of a synchronous generator depends upon its power output and usually limited to a value with in the MVA rating by the capability of its prime mover. Synchronous generators have the capability to produce the maximum MVA output at a specified voltage and power factor (ranging from 0.85 or 0.9 lagging) continuously without overheating. The output of active power is limited by the prime mover capability to a value should be in MVA rating. Based on the three considerations such as armature current limit, field current limit and end region heating limit, the continuous reactive power capability is limited. From the figure (2), the reactive power output may able to reduce active power output capacity of generator, which can also serve as spinning reserve. Therefore it makes implicit financial loss to generators. The reactive power production cost of generator is called opportunity cost which depends upon the real-time balance between load and supply in the market, so it difficult to determine the real value.

The Reactive power cost can be expressed as follows:

$$\text{Cost}(Q_{Gi}) = \text{Cost}(S_{G \max}) - \sqrt{\text{Cost}(S_{G \max}^2 - Q_{Gi}^2)} * k (\$/h) \quad (41)$$

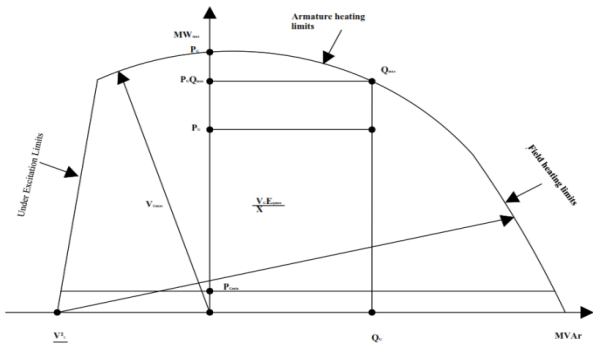


Figure 2. Loading capability curve of generator.

#### IV. STRUCTURE AND OPERATION OF UPFC

Among the available FACTS devices, UPFC is the most advanced FACTS controller that can be used to enhance steady state stability, dynamic stability and transient stability. The UPFC is capable to act over three basic electric system parameters like line voltage, line impedance, and phase angle. UPFC is combination of shunt connected device (STATCOM) and a series connected (SSSC) in the transmission line via its dc link. The UPFC is more flexible, fastest and best featured FACTS device and can be used efficiently and flexibly to optimize line utilization and

increase system reliability and to dampen system oscillations. The UPFC possesses the property of both absorbing and supplying active and reactive power. The schematic diagram of UPFC is shown in Figure 3.

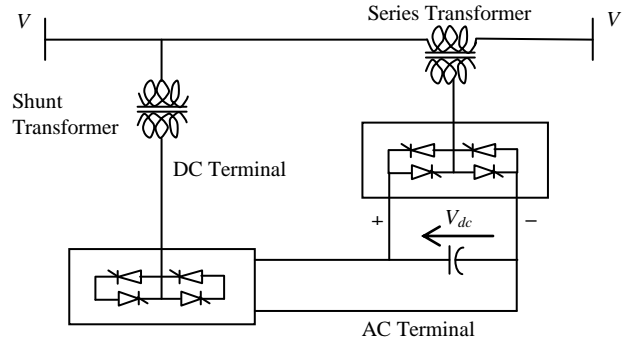


Figure 3. Schematic Diagram of UPFC Controller

It consists of two voltage source converter VSC1 and VSC2 operated from a common dc link provided by a dc storage capacitor which provides dc voltage for the converter operation. One of the two converters is connected in series with the transmission line through a series transformer and the other in parallel with the line through shunt transformer. Hence the real power can able to freely flow in either direction between ac terminals of two VSCs [32]. The rating of UPFC can be set by the power transfer between the series and shunt converters and the rating should be at least as large as the real power exchanged between the two converters. The main function of UPFC is performed by the series converter, which produces the ac voltage of controllable magnitude and phase angle and also injects the voltage at this fundamental frequency in series with the transmission line through a booster transformer. The series converter can be used to increase the transmission capability and exchange the real and reactive power through the series connected transformer. The basic function of shunt converter is to supply or absorb the reactive power demanded by the series converter at the dc terminals and provide independent shunt reactive compensation for the line and also it can be used for local voltage control which improves the system voltage stability.

#### STATIC MODEL REPRESENTATION OF UPFC:

Based on the principle of operation of UPFC and the vector diagram [33], the basic mathematical equations can be written as:

$$V_i' = V_i + V_T \cdot \text{Arg}(I_q) = \text{Arg}(V_i) \pm \pi.2,$$

$$\text{Arg}(I_T) = \text{Arg}(V_i), I_T = \frac{\text{Re}[V_T I_i^*]}{V_i}$$

The Power flow equations from bus-*i* to bus-*j* and similarly from bus-*j* to bus-*i* can be written as

$$S_{ij} = P_{ij} + jQ_{ij} = V_i I_{ij}^* = V_i (jV_j^{-1} B/2 + I_T + I_Q + I_i^{-1})^* \quad (46)$$

$$S_{ji} = P_{ji} + jQ_{ji} = V_j I_{ji}^* = V_j (jV_i B/2 - I_i)^* \quad (47)$$

The above equation is formed with the contribution of active and reactive power flows in the line consider UPFC as,

$$P_{ij} = (V_i^2 + V_T^2)g_{ij} + 2V_iV_Tg_{ij}\cos(\varphi_T - \delta_i) \quad (48)$$

$$-V_jV_T[g_{ij}\cos(\varphi_T - \delta_j) + b_{ij}\sin(\varphi_T - \delta_j)] \\ -V_iV_j[g_{ij}\cos\delta_{ij} - b_{ij}\sin\delta_{ij}] \\ P_{ji} = V_j^2g_{ij} - V_jV_T[g_{ij}\cos(\varphi_T - \delta_j) - b_{ij}\sin(\varphi_T - \delta_j)] \quad (49)$$

$$-V_iV_j[g_{ij}\cos\delta_{ij} + b_{ij}\sin\delta_{ij}] \\ Q_{ij} = -V_iI_q - V_i^2\left(b_{ij} + \frac{b_{sh}}{2}\right) - V_iV_j[g_{ij}\sin\delta_{ij} - b_{ij}\cos\delta_{ij}] \quad (50)$$

$$+ V_iV_T\left[g_{ij}\sin(\varphi_T - \delta_i) + \left(b_{ij} + \frac{b_{sh}}{2}\right)\cos(\varphi_T - \delta_i)\right]$$

$$Q_{ji} = -V_j^2\left(b_{ij} + \frac{b_{sh}}{2}\right) - V_iV_j[g_{ij}\sin\delta_{ij} - b_{ij}\cos\delta_{ij}] \quad (51)$$

$$+ V_jV_T\left[g_{ij}\sin(\varphi_T - \delta_i) + \left(b_{ij} + \frac{b_{sh}}{2}\right)\cos(\varphi_T - \delta_i)\right]$$

The power flow equations (14) to (17) in the model can be replaced with equations (48) to (51) to incorporate the impact of UPFC.

#### COST MODEL OF FACTS DEVICES:

With the introduction of FACTS controllers in the flexible operation of the system, their service needed to be

remunerated and identified because these devices have the capability to change the flow patterns in the network and it causes considerable impact on nodal prices. Hence it should be included in the model of their cost function [29]. Generally there are three basic types of FACTS devices used by the power industry. First type can be characterized as injection of current in shunt model; the second type can be characterized as injection of voltage in series with the line and third type is a combination of current injection in shunt and voltage injection in series pattern [30].

#### COST MODEL OF UPFC:

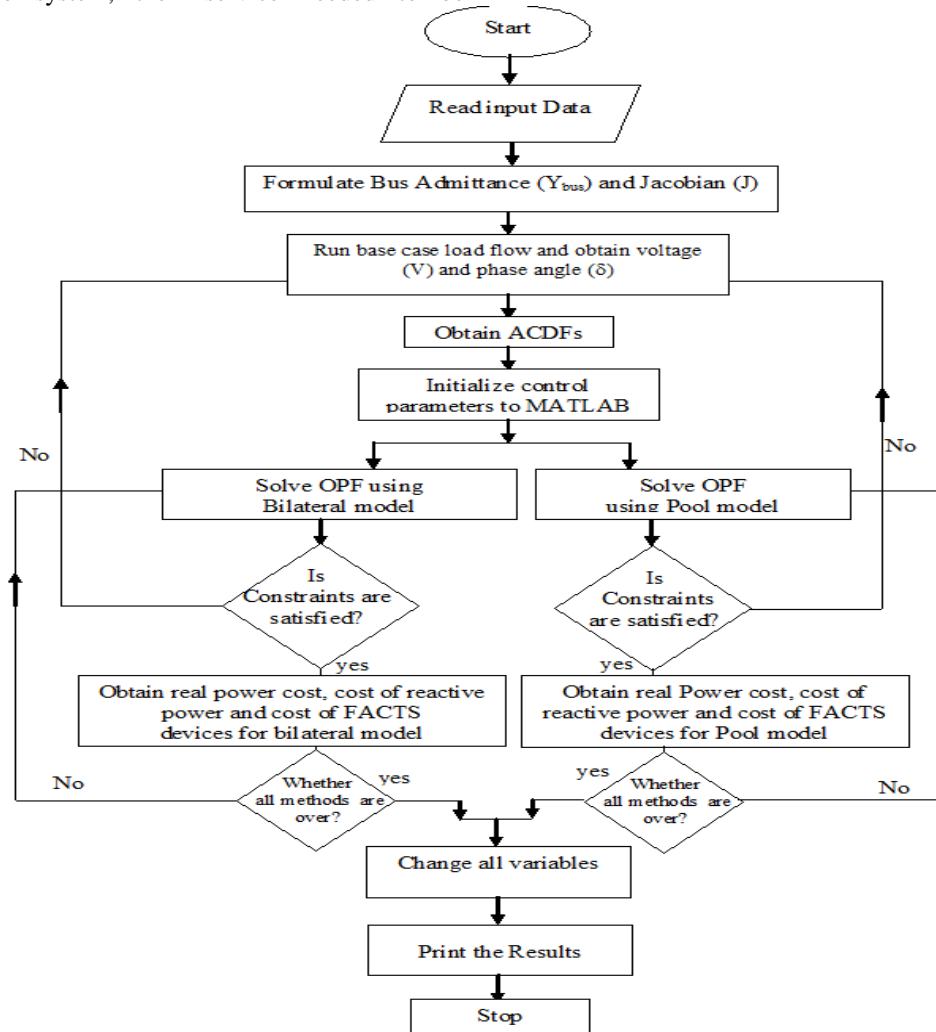
The cost function of UPFC can be considered as [31]:

$$\text{Cost (F)} = 0.0003S - 0.26912S + 188.22 \$ / \text{KVar} \quad (52)$$

Here S is the operating range of the FACTS devices in MVAR. The unit for generation is expressed in US\$/h and for the investment cost of FACTS devices are in US\$ must be unified in to US\$/hour. Generally the FACTS devices will be in service for many years. However, only a part of its life time is considered to regulate the power flow. In this work, five years have been taken in to account to evaluate the cost function of UPFC device. Therefore, the average value of the investment cost is calculated by the following equation:

$$C_1(f) = \frac{C(f)}{8760 * 5} \$ / hr \quad (53)$$

Where C(f) represents the total investment cost of FACTS devices.



## V. SIMULATION RESULTS AND DISCUSSION

The proposed methodology has been applied on a Indian 246-bus NREG system. It is implemented on a computer with Pentium-4, Intel dual core 2.25GHz, 2GB-RAM and simulated in MATLAB 10.0 platform. This test system is adopted from reference [34], comprising 42 generating units, 246 buses and 376 transmission lines. The system line data, bus data and reactor data are considered from the same

reference. The single line diagram of Indian 246-bus NREG system is displayed in figure 5.

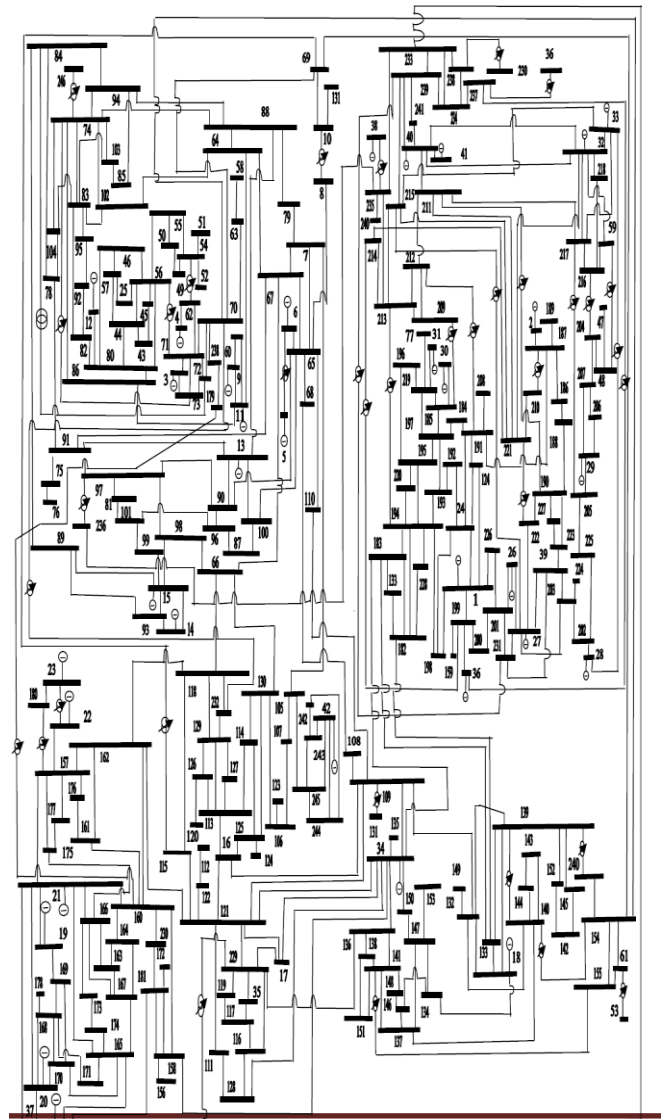
The simulations results are carried out for pool and bilateral model with different cases and the results are characterized as follows:

Case 1: Results without FACTS (UPFC) devices for all methods

Case 2: Results with FACTS (UPFC) devices for all methods  
Bilateral transactions for Indian 246-bus NREG system have been expressed in per unit values and are given in Table 1. The transactions values are considered as additional transactions over and above the already committed pool transactions taken in a system.

**Table I. Values of Bilateral transactions in per unit for Indian 246-bus NREG system**

values of transactions between generator and load bus in per unit	
GD(1,240)=1.5	GD(1,245)=1.0
GD(1,120)=0.2	GD(1,130)=0.3
GD(24,190)=1.2	GD(24,200)=1.0



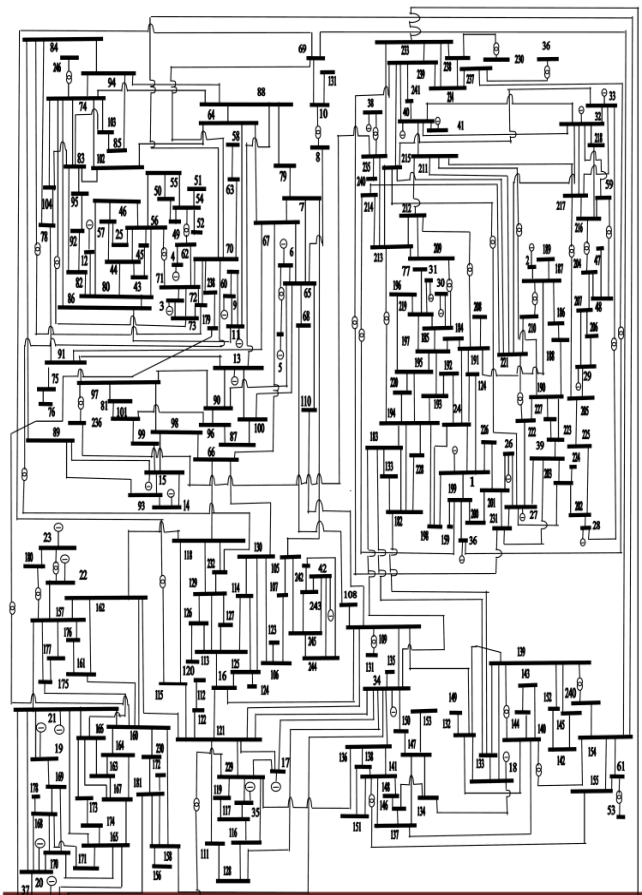
**Figure 5. Single Line Diagram of Indian 246-bus NREG System**

### Case 1: Results for Indian 246-bus NREG System without FACTS Devices

In the first case, the system has not considered the FACTS devices of UPFC. An optimal power flow based non-linear programming has been carried out to calculate the marginal prices of pool and bilateral market model. Table 2 depicts the real power cost, reactive power component cost and total cost of the described methods.

Table 3 elaborates the numerical results of a pool model of an Indian system when it is subjected to a test without FACTS devices. It is observed from the table that the real power cost and reactive power cost is lower for method 1 and method 3 respectively. But the total cost is same for method 1 and method 2. The real power cost of all the methods are compared for without/with FACTS devices and graphically represented in figure (6).

The simulation result of marginal prices of real and reactive power for pool model have been determined by considering three different methods of reactive power cost model of generators and are displayed in Table 4. The reactive power



cost of all the methods are compared for without/with FACTS devices and graphically represented in figure (7).

Table 5 describes the marginal prices of pool and bilateral model for all the three methods. Based on the marginal cost comparisons of the methods, it is proved that the marginal prices of real power at buses are found lesser for bilateral model compared to pool model. It is evident that the changes in the pattern of power flow due to the additional bilateral transactions that take place in the system causes the slight variations in the hybrid market model.

#### Case 2: Results for Indian 246-bus NREG system with FACTS devices

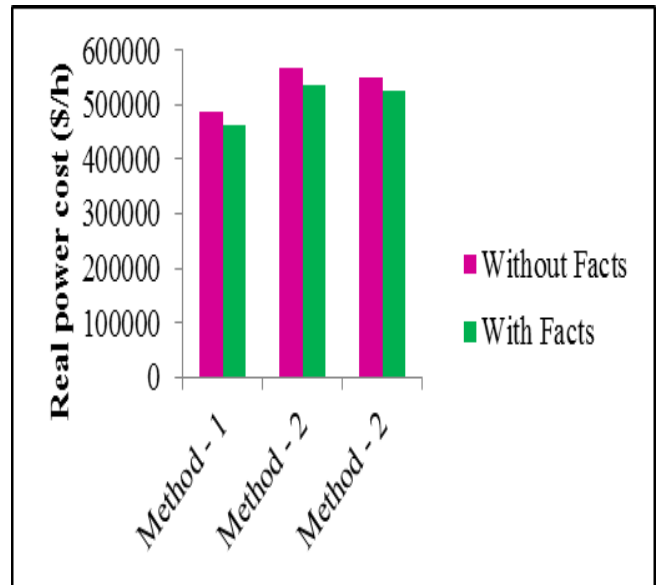
The sudden response of UPFC devices leads to high ability of power system stability and flexibility in managing the power flows. Hence in this case the performance of the proposed method has been improved by installing the UPFC devices. An UPFC has been introduced in the bus number 186 which has a low voltage profile. It is found that the method 1 shows lower real power cost and higher reactive power cost besides the cost function of UPFC is almost same in method 1 and method 2.

Table 6 outlines the real power, reactive power and cost of UPFC based on the three different proposed methods. The marginal cost of three methods has been compared to illustrate its performance. In order to illustrate the performance of UPFC, the marginal cost of three proposed methods have been compared with / without FACTS devices and demonstrated in figure (8).

It is understood from the Table 7, the marginal cost at bus 1 is maximum in pool model and its value is slightly higher than bilateral model. It is proved that the impact of UPFC can be observed at some of the buses and its impact reduces the marginal prices of the three proposed methods and it is found similar at all the buses. Hence in hybrid market model, the additional bilateral transactions takes place in the system changes the line flows patterns which reduce the marginal prices become superior than pool model.

**Table II. Results for NREG 246 – Bus system for without FACTS for Pool Model**

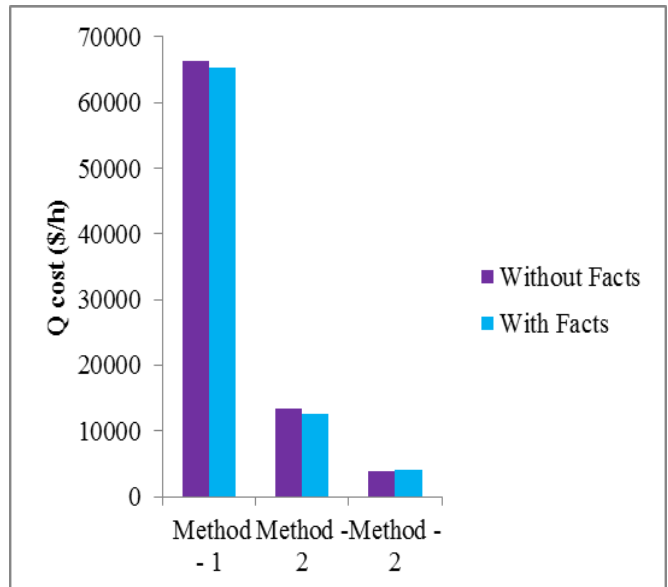
	Method - 1	Method - 2	Method - 3
Real power cost (\$/h)	487074.864	566912.427	549612.012
Q cost(\$/h)	66382.993	13459.345	3842.286
Total cost(\$/h)	553456.857	580376.02	553456.298



**Figure 6: Comparison of Real power cost for all methods of with / Without FACTS**

**Table III. Comparisons of Marginal prices Results for NREG 246 – Bus system for without FACTS for Bilateral Model**

	Method - 1	Method - 2	Method - 3
Real power cost (\$/h)	481396.126	560275.521	543918.618
Q cost (\$/h)	66397.869	12480.496	3876.856
Total cost (\$/h)	547795.075	572756.02	547795.075



**Figure 7: Comparison of Reactive power cost for all methods of with / Without FACTS.**



**Table IV. Results of Locational Marginal prices at few buses of Pool and Bilateral Model for NREG – 246 bus system without FACTS**

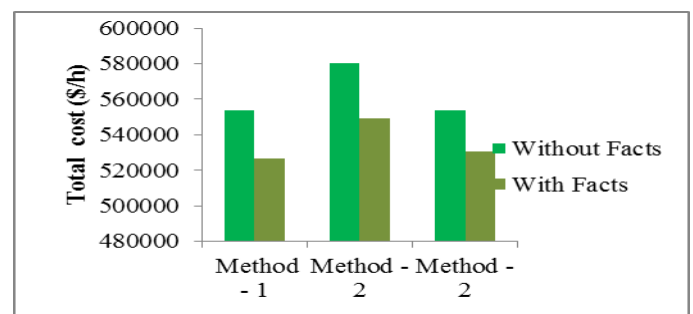
Bus No.	Method-1		Method-2		Method-3	
	Pool	Bilateral	Pool	Bilateral	Pool	Bilateral
1	144.8021	112.8402	144.8021	112.8402	144.8021	112.8402
2	72.371	62.3676	72.371	62.3676	72.371	62.3676
3	88.8005	74.7624	88.8005	74.7624	88.8005	74.7624
4	88.5822	74.6602	88.5822	74.6602	88.5822	74.6602
5	86.7273	72.9655	86.7273	72.9655	86.7273	72.9655
6	86.7273	72.9655	86.7273	72.9655	86.7273	72.9655
7	87.2786	78.9831	87.2786	78.9831	87.2786	78.9831
8	85.8943	72.1554	85.8943	72.1554	85.8943	72.1554
9	86.7826	73.0206	86.7826	73.0206	86.7826	73.0206
10	86.2181	72.4188	86.2181	72.4188	86.2181	72.4188
11	87.6176	73.7195	87.6176	73.7195	87.6176	73.7195
12	88.1304	74.1767	88.1304	74.1767	88.1304	74.1767
13	86.2184	73.0531	86.2184	73.0531	86.2184	73.0531
14	87.3827	73.4698	87.3827	73.4698	87.3827	73.4698
15	87.4693	73.5305	87.4693	73.5305	87.4693	73.5305
16	88.2086	74.0009	88.2086	74.0009	88.2086	74.0009
17	87.6657	73.4426	87.6657	73.4426	87.6657	73.4426
18	90.1125	75.1364	90.1125	75.1364	90.1125	75.1364
19	82.8538	70.9566	82.8538	70.9566	82.8538	70.9566
20	82.5389	71.3324	82.5389	71.3324	82.5389	71.3324
21	83.2873	68.7928	83.2873	68.7928	83.2873	68.7928
22	80.5355	69.0434	80.5355	69.0434	80.5355	69.0434
23	80.8604	97.737	80.8604	97.737	80.8604	97.737
24	123.037	73.8127	123.037	73.8127	123.037	73.8127
24	87.6669	72.8854	87.6669	72.8854	87.6669	72.8854
25	84.2682	70.2245	84.2682	70.2245	84.2682	70.2245
26	84.4682	72.3892	84.4682	72.3892	84.4682	72.3892
27	90.1123	73.5504	90.1123	73.5504	90.1123	73.5504
28	82.4532	75.0077	82.4532	75.0077	82.4532	75.0077
29	83.7865	74.1671	83.7865	74.1671	83.7865	74.1671
30	82.2867	72.8865	82.2867	72.8865	82.2867	72.8865
31	82.5432	72.1123	82.5432	72.1123	82.5432	72.1123
32	118.976	70.1057	118.976	70.1057	118.976	70.1057
33	91.1008	71.1642	91.1008	71.1642	91.1008	71.1642
34	86.9174	72.1465	86.9174	72.1465	86.9174	72.1465
35	120.348	73.8126	120.348	73.8126	120.348	73.8126
36	84.7653	72.1767	84.7653	72.1767	84.7653	72.1767
37	88.9002	73.0531	88.9002	73.0531	88.9002	73.0531
38	90.0045	73.7371	90.0045	73.7371	90.0045	73.7371
39	83.6754	70.2368	83.6754	70.2368	83.6754	70.2368
40	82.4862	71.4465	82.4862	71.4465	71.4465	71.4465

**Table V. Results for NREG 246 – Bus system for with FACTS for pool model**

	Method - 1	Method - 2	Method - 3
Real power cost (\$/h)	461464.282	536684.865	526754.342
Q cost (\$/h)	65298.443	12567.230	4018.667
Cost of UPFC (\$/h)	2.225	2.225	3.176
Total cost (\$/h)	526764.95	549254.32	530776.185

**Table VI. Comparisons of Marginal prices Results for NREG 246 – Bus system for with FACTS for Bilateral Model**

	Method - 1	Method - 2	Method - 3
Real power cost (\$/h)	471568.886	538880.786	523334.745
Q cost (\$/h)	674324.621	10566.112	3432.889
Cost of UPFC (\$/h)	2.168	4.899	2.168
Total cost (\$/h)	525643.990	551123.90	516543.967



**Figure 8. Comparisons of total Cost for all Methods of with / Without FACTS**

**Table VII. Results of Locational Marginal prices at few buses of Pool and Bilateral Model for NREG – 246 bus system with FACTS**

Bus No.	Method-1		Method-2		Method-3	
	Pool	Bilateral	Pool	Bilateral	Pool	Bilateral
1	140.6443	110.6103	140.6443	110.6103	140.6443	110.5463
2	2.1234	61.2034	2.1234	61.2034	2.1234	61.1024
3	8.2345	73.6852	8.2345	73.6852	8.2345	73.4356
4	8.2213	73.1267	8.2213	73.1267	8.2213	73.0976
5	5.8896	72.2255	5.8896	72.2255	5.8896	72.1123
6	5.8896	72.2255	5.8896	72.2255	5.8896	72.1123
7	7.6545	73.0005	7.6545	73.0005	7.6545	73.0002
8	4.9889	71.3323	4.9889	71.3323	4.9889	71.1646
9	84.3678	71.0897	84.3678	71.0897	84.3678	71.0643
10	86.1008	72.3467	86.1008	72.3467	86.1008	72.2361
11	86.1008	71.8968	86.1008	71.8968	86.1008	71.6879
12	87.2112	71.2352	87.2112	71.2352	87.2112	71.1765
13	87.1042	73.1143	87.1042	73.1143	87.1042	73.1043
14	84.7939	74.0123	84.7939	74.0123	84.7939	74.0067
15	87.9988	72.2214	87.9988	72.2214	87.9988	72.1123
16	88.0876	72.3675	88.0876	72.3675	88.0876	72.2654
17	85.3342	73.1268	85.3342	73.1268	85.3342	73.0234
18	82.6075	73.1054	82.6075	73.1054	82.6075	73.0078
19	82.4167	74.1896	82.4167	74.1896	82.4167	74.0896
20	82.5543	69.9015	82.5543	69.9015	82.5543	69.6065
21	79.4437	70.1045	79.4437	70.1045	79.4437	70.1005
22	79.2004	70.2435	79.2004	70.2435	79.2004	70.1123
23	21.6785	67.8202	21.6785	67.8202	21.6785	67.4327
24	87.9078	68.1289	87.9078	68.1289	87.9078	68.1133
24	82.1776	97.1643	82.1776	97.1643	82.1776	97.0088
25	82.1776	72.6607	82.1776	72.6607	82.1776	72.66
26	83.0097	71.2668	83.0097	71.2668	83.0097	71.2662
27	112.6787	72.4554	112.6787	72.4554	112.6787	72.3356
28	122.134	73.3389	122.134	73.3389	122.134	73.0643
29	78.2345	73.1682	78.2345	73.1682	78.2345	73.0078
30	77.9908	96.5543	77.9908	96.5543	77.9908	96.2343
31	82.4455	93.8926	82.4455	93.8926	82.4455	93.7761
32	83.6677	75.4166	83.6677	75.4166	83.6677	75.4018
33	84.1122	75.4488	84.1122	75.4488	84.1122	74.2214
34	122.889	74.3325	122.889	74.3325	122.889	71.0001
35	87.987	71.0008	87.987	71.0008	87.987	70.3636
36	82.776	70.6576	82.776	70.6576	82.776	68.4292
37	88.445	68.4698	88.445	68.4698	88.445	67.2424
38	86.244	67.4545	86.244	67.4545	86.244	72.3356
39	84.2816	72.4305	84.2816	72.4305	84.2816	72.4305
40	83.1432	71.6789	83.1432	71.6789	83.1432	71.6789

## VI. CONCLUSION

In this work, an attempt has been made for determination of marginal price for real and reactive power with the reactive power's cost model function. The introduction of FACTS devices and its cost model endeavored to find their impact on real and reactive power nodal price at each bus is presented. The marginal prices of real and reactive power for pool and bilateral models have been obtained and compared for the justification. The inception of FACTS devices plays a crucial role in defining the marginal prices. From the analysis, it is proved that the marginal prices are found lower in bilateral model when compared with pool model. This is because the change in flow pattern through additional bilateral transactions. Based on the results, it is concluded that reactive

power cost component have considerable effect on nodal price determination of real and reactive power at each bus.

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## REFERENCES

- [1] M.L. Baughman and S.N.Siddiqi, "Real Time Pricing of Reactive Power: Theory and Case Study Results," *IEEE Transactions on Power Systems*, Volume 6, No.1, pp 23-29, 1991.
- [2] S.Hao and A.Pepalexopolous, "A Reactive Power Pricing and Management," *IEEE Transactions on Power Systems*, Volume 12, No.1, pp 95-104, 1997.

- [3] Ro K, "Calculation of Reactive Power Charges Under Competition of Electric Power Industries", *Electrical Engineering*, 85, pp 169-175, 2003.
- [4] Alvarado F, "Reactive Power as an Identifiable Ancillary Service", *Alberta report*, 2003.
- [5] Y.Z.Li and A.K. David, "Wheeling Rates of Reactive Power Under Marginal Cost Theory", *IEEE Transactions on Power Systems*, Volume 10, No. 3, 1993.
- [6] Chung CY, Cheng TS, Yu CW, Lin Xi, "Cost Based Reactive Power Pricing With Voltage Security Consideration in Restructured Power Systems", *Electric Power System Research*, No, 70, Issue, 1, pp 85-92, 2004.
- [7] J.B.Gill, T.G.S.Roman, J.J.A.Rioa and P.S.Martin, "Reactive Power Pricing: A Conceptual Framework for Renumeration and Charging Procedures", *IEEE Transactions on Power Systems*, Volume 15, No.2, pp483-489, 2000.
- [8] D.Chattopadhyaya, K.Bhattacharya and J. Parikh, "Optimal Reactive Power Planning and Its Spot Pricing: An integrated Approach", *IEEE Transactions on Power Systems*, Volume 10, No.4, pp 2014-2019, 1995.
- [9] J.Y.Choi, S.Rim and J. Park, "Optimal Real Time Pricing of Real and Reactive Powers", *IEEE Transactions on Power Systems*, Volume 13, No.4, pp 1226-1231, 1998.
- [10] M.Muchayi and M.E.El-Harwary, "A Summary of Algorithms in Reactive Power Pricing", *Electric Power and System Research*, Volume 21, pp 119-124, 1999.
- [11] M.J.Rider and M.L.Paucar, "Application of nonlinear Reactive Power Pricing Model for Competitive Electric Markets", *IEEE Proceedings-Generation, Transmission, Distribution*, Volume 151, No.3, pp 407-414, 2004.
- [12] V.L.Paucer and M.J.Rider, "Reactive Power Pricing in Deregulated Electrical Markets Using Methodology Based on Theory of Marginal Cost", *Proceedings of IEEE PES General Meeting*, Vancouver, 2001.
- [13] J.J.Paserba, "How FACTS Controllers Benefits AC Transmission System", *IEEE Transmission on Power Engineering Society*, Volume 3, pp 949-956, 2003.
- [14] L.J.Cai and Istvan Erlich, "Optimal Choice and Allocation of FACTS Devices Using Genetic Algorithms", *Proceedings of ISAP, Intelligent Systems Application To Power Systems*, Lemons, 31-August-3 September, 2003.
- [15] E.J.De Olivera, J.W.M.Lima and J.L.R.Pereira, "Flexible AC Transmission Devices Allocation and Transmission Pricing", *Electric Power and Energy Systems*, Volume 21, No.2, pp 111-118, 1999.
- [16] S.C. Srivastava and K.Verma, "Impact of Facts Devices on Transmission Pricing in A Deregulated Market", *International Conference on Electric Utility Deregulation and Restructuring and Power Technologies, Drpt, London*, pp 642-648, April 4-7, 2000.
- [17] G.B.Shrestha and W.Feng, "Effects of Series Compensation on Spot Price Market", *Electrical Power and Energy Systems*, Volume 27, No.5-6, pp 428-436, 2005.
- [18] F.D.Galiana and M.Ilic, "A Mathematical Framework for The Analysis and Management of Power Transactions Under Open Access", *IEEE Transactions on Power Systems*, Volume 13, No.2, pp 681-687, 1998.
- [19] Singh, S. N. and David, A. K., "Placement of FACTS Devices in Open Power Market" in *Proc. 5th Conference on Advances in Power System Control, Operation and Management, APSCOM*, pp. 173-177, Hong Kong, 2000.
- [20] A.Kumar and Punit Kumar, "Locational Marginal Prices With SVC in Indian Electricity Market", *International Journal of Engineering Science and Technology*, Volume 3, No.3, pp 9-24, 2011.
- [21] M.Shahidehpour, H.Yamin and Z.Li, "Market Operations in Electric Power Systems: Forecasting, Scheduling & Risk Management", *John Wiley & Sons Limited*, 2002.
- [22] Kankare Bhattacharya, Math H.J.Bollen, Jaap E.Daelder, "Operation of Restructured Power System", *Kluwer Academic Publishers*, 2001.
- [23] Hamoud G, "Feasibility Assessment of Simultaneous Bilateral Transactions in A Deregulated Environment", *IEEE Transactions on Power Systems*, Volume 15, No.1, pp 22-26, 2000.
- [24] Cheng J.W.M, Galiana F.D and M.E.Gills, D.J., "Studies of Bilateral Contracts With Respect To Steady State Security in a Deregulated Environment", *IEEE Transactions on Power Systems*, Volume 15, No.3, pp 1020-1025, 1998.
- [25] Chanana S and Kumar A, "Marginal Prices of Real and Reactive Power in Hybrid Electricity Markets", *Journal of CBIP*, January 2010.
- [26] Dai Y, Ni, Y.X.Wen, F.S and Han, Z.X, "Analysis of Reactive Power Under Deregulation", in *Proceedings of IEEE PES*, Volume 1, Issue 1, pp 2162-2167, 2000.
- [27] Y.Zhao, M.R.Irving and Y.Song, "A Cost Allocation and Pricing Method of Reactive Power Services in The New Deregulated Electricity Market Environment", *Proceedings of IEEE Transmission and Distribution Conference*, Dalian, August 2005.
- [28] S.Hasanpour, R.Ghazi and M.H.Javidi, "A New Approach for Cost Allocation and Reactive Power Pricing in a Deregulated Environment", *Electrical Engineering*, 91, pp 27-34, 2009.
- [29] Y.H. Song and A. Johns, *Flexible AC Transmission systems (FACTS)*. London, U.K.: IEE, 1999.
- [30] Ashwani Kumar, "Reactive Power and FACTS Cost Model's Impact on Nodal Pricing in Hybrid Electricity Markets", *Smart Grid and Renewable Energy*, Volume 1, Issue 2, pp 230-244, 2011.
- [31] Z.Huang, Y.Ni, C.Shen, F.F.Wu, S.Chen and B.Zhang, "Application of UPFC in Interconnected Power Systems- Modeling, Interfacing, Control Strategy and Case Study", *IEEE Transactions on Power System*, Volume 15, pp.817-824, 2000.
- [32] A.K.Sharma and S.Chanana, "New Multiobjective Optimization Problem for Secure Bilateral Transaction Determination With Upfc in Hybrid Electricity Markets", *Electric Power Components and Systems*, Volume 36, No.6, pp 555-574, 2008.
- [33] G.Shaoyun, "Optimal Power System Operation and Control incorporating FACTS Devices", Ph.D. Thesis, *The Hong Kong University*, August 1998
- [34] Indian 246-Bus NREG System - A Report From Department of Electrical Engg, IIT, Kanpur.

## NOMENCLATURE

$N_g$	Set of generators
$N_b$	Number of buses in the system
$N_d$	Number of load buses
$P_{gi}$	Active power pool generator- $i$
$C_i$	Fuel cost of pool generator- $i$
$a_{gi}, b_{gi}, c_{gi}$	Cost coefficients in \$/h, \$/MWh, \$/MWh <sup>2</sup>
$P_i$	Real power injection at bus- $i$
$Q_i$	Reactive power injection at bus- $i$
$P_{gi}, Q_{gi}$	Real and reactive power generation at bus- $i$
$I_a$	Armature current of generator
$P_{di}, Q_{di}$	Real and reactive power demand at bus- $i$
$V_i$	Voltage magnitude at bus- $i$
$\delta_i$	Voltage angle at bus- $i$
$P_{gi, \min}, P_{gi, \max}$	Minimum and maximum real power generation limit
$Q_{gim, \min}, Q_{gim, \max}$	Minimum and maximum reactive power generation limit
$S_{gmax}$	Maximum apparent power
$V_{i, \min}, V_{i, \max}$	Upper and lower voltage magnitude limit
$\delta_{\min}, \delta_{\max}$	Upper and lower voltage angle limit
$S_{ij}, S_{ij} < S_{ij}^{\max}$	Line flow limit
$ACDF$	The distribution factors
$T$	Transaction Matrix
$T_{ij}$	Bilateral transactions between seller and buyer bus $i$ and $j$
$T_{ij}^{\max}$	Maximum transaction amount;
$x$	State vector of variables $V, \delta$ ;
$u$	Control parameters, $P_{gi}, Q_{gi}, P_{gb}, P_{gp}$ ;
$p$	Fixed parameters $P_{di}, P_{db}, P_{dp}, Q_d, T_{ij}$ ;
$\zeta_{UPFC}^{\text{int}}$	An integer variable showing absence or presence of FACTS devices with integer values $\{0, 1\}$ ;
$GD$	Bilateral Transaction Matrix
$P_{DB}$	Vector of Bilateral Demand
$P_{DP}$	Vector of Pool Demand
$P_{GB}$	Vector of Bilateral Generation
$P_{GP}$	Vector of Pool Generation